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OBSERVATIONS ON APPROXIMATE INTEGRATIONS,(U)

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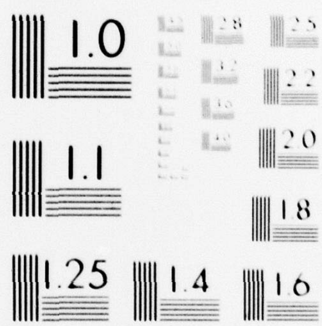

MICROCOPY RESOLUTION TEST CHART

OBSERVATIONS ON APPROXIMATE INTEGRATIONS*

by

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Extended Abstract

In this presentation ~~we explore~~ a class of integration strategies are explored that fall in between the two extremes of symbolic integration and numerical quadrature, which are, respectively, aimed at the computer generation of answers in the form of exact expressions and numerical values. ~~We shall first discuss~~ the theoretical advances in symbolic integration are discussed first, as motivation to the following, then ~~examine~~ three major contexts of applications with attendant case studies are examined, and finally ~~explore~~ four possible types of strategies for approximate integration are explored. In particular we shall comment on the feasibility and adequacy (or inadequacy) of MACSYMA for implementing these strategies.

We begin with theoretical discussions. In this aspect we have discerned two major paradigms of strategies, which we label the "pattern-recognition paradigm" and the "problem-solving paradigm". These labels, though far from perfect, are chosen to indicate the emphasis only. In the former class we include, for example, Risch's algorithm, (Ref. 1) and Moses' new approach based on extension operators (Ref. 2). We believe these strategies to be particularly characterized by the search for algorithmic techniques to recognize that certain expressions or operators belong to some specified classes. The problem solving paradigm is obviously inherited from heuristic strategies of artificial intelligence. In this latter class we include, for example, Wang's definite integrations (Ref. 3) and our elliptic integrations (Ref. 4). All these theoretical strategies suffer from practical limitations of one kind or another. Notebly among these are the multivariate factorization problem, the optimal selection of

input vis-a-vis output class of expressions and intelligent choice of contours for definite integration. The optimal selection needs particular elaboration here. Take for example the integration of rational functions. It is easy to devise an efficient algorithm to decide if a given rational function can be integrated in terms of rational functions. But such algorithm would be of extremely limited interest because it would return a negative answer for most input expressions, such as something as simple as 1/(x+1). The addition of one 'new' function (logarithm) in the output class dramatically expands the problem-solving horizon. On the other hand, we obviously cannot carry this to the other extreme of choosing a large number of new functions, lest the result be next to worthless. All these discussions, however, force us to consider what we mean by 'usefulness' of an output expression, which in turn leads us to considering three major contexts of applications.

At this Laboratory we have been associated with an applied mathematics group which provides consultation and support to a diversity of engineers and scientists. Although our picture is still somewhat limited, it does give us an indication of the major contexts in which integration tools are considered necessary or useful. The first is the usual exploratory context, where a scientist or engineer encounters isolated integrals which he needs to tackle. Here he typically wants closed form solution, but often settles for an approximate answer. The need here is based on the motivation to "do something with" the result, that is, to either study its dependency on some parameters or on some other mathematical operations. The second context revolves around multiple integration. Here the goal is usually numerical evaluation, but one is interested in reducing the multiplicity of integration as much as possible, because multiple quadrature is costly both in computing time and accuracy. The third context concerns multi-parameter studies, where the integral depends on a number of parameters, thus making

numerical results difficult, if not impossible to interprete. For example, if the integral is a function of six parameters, the numerical result would require a six-dimensional table or six-dimensional hypersurface to represent. In all these contexts of applications, current technology forces an investigator to take either alternative of the two extremes of numerical versus analytic results (with some exceptions to be mentioned later). It is fair to say that most "real life" problems are non-elegant in nature and analytic results are difficult and unlikely to come by. For example, a polynomial of 5th degree whose coefficients are derived from data or other computations can seldom be factorized. In most non-trivial algorithms of integration this fundamental limitation is often fatal, because they involve, in one form or another, partial fraction decomposition which depends on factorization. All these discussions point to the need of a compromising approach between the extremes of numerical and exact integration. Such an approach (let us call it approximate integration), is resorted by scientists and engineers in isolated instances, but has not been investigated as a possible general purpose tool in the sense of a quadrature scheme or a symbolic integration algorithm. The inportant point to stress is that the approximate approach is intended to yield an output that is an expression, rather than a table of numbers. At this stage we have examined four broad categories of such approximate schemes. The first constitutes in the approximation of the integrand by a set of basis functions such as polynomials or splines. There have been some isolated applications using such approximation, for example, in finite element analysis. The second may be labelled interpolatory scheme. Here the spirit is analogous to the derivation of quadrature schemes, i.e., by approximating the integrand by some interpolation formula and then integrating term by term. The third approach is based on a reduction of transcendence of the integrand. For example, if it is intended to integrate the sine

of a polynomial, one may approximate the sine by a rational function and thereby reduce the entire problem to one of integration of rational functions. The last is to compute a parameterized set of integrals by quadrature and then approximate the answer by some basic functions. This approach can hardly be considered under the umbrella of integration (it is more of a curve or surface fitting problem), but it may turn out to be very useful in some contexts. In the presentation we shall provide a concrete example for each approach and discuss the MACYSMA relevance to each. Though we do not have a coherent theory behind each, we believe this investigation is a modest beginning of approaches of practical significance.

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